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Entropy function for non-extremal D1D5 and D2D6NS5-branes

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Abstract: We apply the entropy function formalism to non-extremal $D1D5$ and $D2D6NS5$-branes whose throat approximation is given by the Schwarzschild black hole in $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^2 \times S^1 \times T^4$, respectively. We find the Bekenstein-Hawking entropy and the $(\alpha')^3 R^4$ corrections from the value of the entropy function at its saddle point. While the higher derivative terms have no effect on the temperature, they decrease the value of the entropy.

Keywords: Black Holes in String Theory, D-branes.

Contents

1. Introduction

The black hole attractor mechanism has been an active subject over the past few years in string theory. This is originated from the observation that there is a connection between the partition function of four-dimensional BPS black holes and partition function of topological strings [1]. This mechanism states that in the extremal black hole backgrounds the moduli scalar fields at horizon are determined by the charge of black hole and are independent of their asymptotic values. One may study the attractor mechanism by finding the effective potential for the moduli fields and examining the behavior of the effective potential at its extremum. This extremum should be a local minimum for extremal black holes. The entropy of black hole is then given by the value of the effective potential at its minimum. Using this, the entropy of some extremal black holes has been calculated in [2].

Recently, it has been proposed by A. Sen that the entropy of a specific class of extremal black holes in higher derivative gravity can be calculated using the entropy function formalism [4]. According to this formalism, the entropy function for the black holes that their near horizon is $AdS_2 \times S^{D-2}$ is defined by integrating the Lagrangian density over S^{D-2} for a general $AdS_2 \times S^{D-2}$ background characterized by the size of AdS_2 and S^{D-2}, and taking the Legendre transform of the resulting function with respect to the parameters labeling the electric fields. The result is a function of moduli scalar fields as well as the sizes of AdS_2 and S^{D-2}. The values of moduli fields and the sizes are determined by extremizing the entropy function with respect to the moduli fields and the sizes. Moreover, the entropy is given by the value of the entropy function at the extremum.[1] Using this method the entropy of some extremal black holes have been found in [4–6].

[1] It is assumed that in the presence of higher derivative terms there is a solution whose near horizon geometry is $AdS_2 \times S^{D-2}$. In the cases that the higher derivative corrections modify the solution such that the near horizon is not $AdS_2 \times S^{D-2}$ anymore, one cannot use the entropy function formalism. In those cases one may use the Wald formula [3] to calculated the entropy directly.

The above discussion does not indicate that the entropy function should have local minimum at the near horizon. In fact, it has been shown in [7] that the entropy function has a saddle point at the near horizon of extremal black holes. One may then conclude that the entropy function formalism should not be something specific for the extremal black holes. Indeed, it has been shown in [8, 7] that the entropy function formalism works for some non-extremal black hole/branes at the supergravity level. It has been speculated in [7] that the entropy function formalism works for the non-extremal black holes/branes whose near horizons are some extension of AdS space, e.g., Schwarzschild black hole in AdS.

The non-extremal black branes that have been studied in [7] are $D3$, $M2$ and $M5$-branes whose near horizon geometries are Schwarzschild black hole in AdS_{p+2} where $p = 3, 2$ and 5, respectively. When higher derivative corrections are included, however, the near horizon geometry is not the Schwarzschild black hole in AdS_{p+2} anymore. Consequently, the entropy function formalism does not work for these cases when one considers the higher derivative terms. In this paper, we would like to study the non-extremal black hole/brane solutions that the higher derivative terms respect the symmetry of the tree level solutions. Consider the non-extremal $D1D5$ and $D2D6NS5$-branes. The near horizon (throat approximation) of their tree level geometries are the Schwarzschild black hole in AdS_3. Moreover, in these cases, the higher derivative terms of the effective action respect the symmetry of the supergravity solution. In fact, the Schwarzschild black hole in AdS_3 is the BTZ black hole [9] in which the inner horizon $\rho_- = 0$. On the other hand, it is known that the BTZ black hole is an exact solution of the string theory [10]. So one expects that the entropy function formalism works for the non-extremal $D1D5$ and $D2D6NS5$-branes even in the presence of the higher derivative terms.

An outline of the paper is as follows. In section 2, we review the non-extremal $D1D5$ and $D2D6NS5$ solutions of the effective action of type II string theory. In sections 3, using the entropy function formalism we derive the Bekenstein-Hawking entropy of $D1D5$-branes in terms of the temperature of black branes. We show that the entropy is given by the entropy function at its saddle point. In subsection 3.1 we show that the higher derivative terms respect the symmetries of the solution at the tree level and the entropy function formalism works in the presence of the higher derivative terms. Using this we find the entropy as the saddle point of the entropy function. As a double check, we also calculate the entropy using the Wald formula directly and find exact agreement with the result from the entropy function formalism. In section 4, we repeat the calculations for $D2D6NS5$-branes. We shall show that, in both cases, the higher derivative terms do not modify the tree level temperature, however, the entropy decreases with respect to the Bekenstein-Hawking entropy.

2. Review of the non-extremal solutions

In this section we review the non-extremal $D1D5$ and $D2D6NS5$-branes solutions of the effective action of type II string theory. The two-derivatives effective action in the string frame is given by

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \Big\{ e^{-2\phi} \Big(R + 4(\partial\phi)^2 - \frac{1}{12} H^2_{(3)} \Big) - \frac{1}{2} \sum \frac{1}{n!} F^2_{(n)} + \cdots \Big\}, \tag{2.1}$$

where ϕ is the dilaton, $H_{(3)}$ is NS-NS 3-form field strength, and $F_{(n)}$ is the electric R-R n-form field strength where $n = 1, 3, 5$ for IIB and $n = 2, 4$ for type IIA theory. In above equation, dots represent fermionic terms in which we are not interested. The effective action includes a Chern-Simons term which is zero for the $D1D5$ and $D2D6NS5$ solutions. Moreover, for these solutions $F_{(n)} = dC_{(n-1)}$. The 5-form field strength tensor is self-dual, hence, it is not described by the above simple action. It is sufficient to adopt the above action for deriving the equations of motion, and impose the self-duality by hand.

The non-extremal $D1D5$-branes solution of the IIB effective action when $D1$-branes are along the compact (z) direction (S^1) and $D5$-branes along the compact (z, x_1, x_2, x_3, x_4) directions ($S^1 \times T^4$) is given by the following, (see e.g. [11]):

$$\begin{aligned} ds^2 &= (f_1 f_5)^{-\frac{1}{2}} \Big(-f dt^2 + dz^2 \Big) + (f_1 f_5)^{\frac{1}{2}} \Big(\frac{dr^2}{f} + r^2 (d\Omega_3)^2 \Big) + \Big(\frac{f_1}{f_5} \Big)^{\frac{1}{2}} \sum_{i=1}^{4} dx_i^2 \,, \\ e^{-2\phi} &= \frac{f_5}{f_1} \,, \qquad C_{tz} = \Big(\frac{1}{f_1} - 1 \Big) \,, \qquad C_{tzx_1 \cdots x_4} = \Big(\frac{1}{f_5} - 1 \Big) \,, \end{aligned} \tag{2.2}$$

where

$$f_1 = 1 + \frac{Q_1}{r^2} \,, \qquad f_5 = 1 + \frac{Q_5}{r^2} \,, \qquad f = 1 - \frac{r_0^2}{r^2} \,. \tag{2.3}$$

The above solution is the D1D5P solution [11, 14] in which the amount of left and right moving momenta, propagating in the compact direction z, is chosen to be equal, i.e., $\sigma = 0$ in the notation [14].

For $r_0 = 0$ we obtain the extremal solution, depending on the two parameters Q_1 and Q_5 which are related to the number of D-branes. For $r_0 \neq 0$ a horizon develops at $r = r_0$. The near horizon geometry which is described by a throat, can be found by using the throat approximation where $r \ll Q_1$ and $r \ll Q_5$. In these limits the non-extremal solution becomes

$$\begin{aligned} ds^2 &= \frac{r^2}{\sqrt{Q_1 Q_5}} \Big\{ -\Big(1 - \frac{r_0^2}{r^2} \Big) dt^2 + dz^2 \Big\} + \frac{\sqrt{Q_1 Q_5}}{r^2} \Big(1 - \frac{r_0^2}{r^2} \Big)^{-1} dr^2 \\ &\qquad + \sqrt{Q_1 Q_5} (d\Omega_3)^2 + \sqrt{\frac{Q_1}{Q_5}} \sum_{i=1}^{4} dx_i^2 , \\ e^{-2\phi} &= \frac{Q_5}{Q_1} \,, \qquad F_{rtz} = 2\frac{r}{Q_1} \,, \qquad F_{rtzx_1 \cdots x_4} = 2\frac{r}{Q_5} \,. \end{aligned} \tag{2.4}$$

The geometry is the product of $S^3 \times T^4$ with the Schwarzschild black hole in AdS_3.

The non-extremal $D2D6NS5$-branes solution of the IIA effective action when $D2$-branes are along the compact (z, x_1) directions ($S^1 \times S'^1$), $D6$-branes along the compact $(z, x_1, x_2, x_3, x_4, x_5)$ directions ($S^1 \times S'^1 \times T^4$) and $NS5$-branes along the compact

(z, x_2, x_3, x_4, x_5) directions ($S^1 \times T^4$) is given by the following (see e.g. [13]):

$$\begin{aligned}
ds^2 &= (f_2 f_6)^{-\frac{1}{2}}\Big(-f dt^2 + dz^2\Big) + f_5 (f_2 f_6)^{\frac{1}{2}}\left(\frac{dr^2}{f} + r^2 (d\Omega_2)^2\right) \\
&\quad + f_5 (f_2 f_6)^{-\frac{1}{2}} dx_1^2 + \left(\frac{f_2}{f_6}\right)^{\frac{1}{2}} \sum_{i=2}^{5} dx_i^2 , \\
e^{-2\phi} &= f_5^{-1} f_6^{\frac{3}{2}} f_2^{-\frac{1}{2}} , \qquad C_{tzx_1} = \coth\alpha_2 \left(\frac{1}{f_2} - 1\right) + \tanh\alpha_2 , \\
H_{x_1 ij} &= \epsilon_{ijk}\partial_k f_5' , \qquad (dA)_{ij} = \epsilon_{ijk}\partial_k f_6' , \qquad i = 6, 7, 8 ,
\end{aligned} \tag{2.5}$$

where

$$f = 1 - \frac{r_0}{r} , \;\; f_n = 1 + \frac{r_0 \sinh^2\alpha_n}{r} , \;\; f_n' = 1 + \frac{r_0 \sinh\alpha_n \cosh\alpha_n}{r} , \qquad n = 2, 5, 6 . \tag{2.6}$$

The above solution is the D2D6NS5P solution [13, 14] in which the amount of left and right moving momenta, propagating in the compact direction z, is chosen to be equal, i.e., $\alpha_p = 0$ in the notation [14].

For $r_0 \to 0$ one obtains the extremal solution by sending $\alpha_n \to \infty$ such that $r_0 \sinh^2\alpha_n \equiv Q_n$ is kept fixed. The extremal solution then depends on the three parameters Q_2, Q_5 and Q_6 which are related to the number of D-branes. For $r_0 \neq 0$ a horizon develops at $r = r_0$. The near horizon geometry which is described by a throat can be found by using the throat approximation where $r \ll Q_n$ and $Q_n \equiv r_0 \sinh^2\alpha_n$. In this limit $\cosh\alpha_n \sim \sinh\alpha_n$ and the non-extremal solution becomes

$$\begin{aligned}
ds^2 &= \frac{\rho^2}{4Q_5\sqrt{Q_2Q_6}}\left\{-\left(1 - \frac{\rho_0^2}{\rho^2}\right) d\tau^2 + dy^2\right\} + \frac{4Q_5\sqrt{Q_2Q_6}}{\rho^2}\left(1 - \frac{\rho_0^2}{\rho^2}\right)^{-1} d\rho^2 \\
&\quad + Q_5\sqrt{Q_2Q_6}(d\Omega_2)^2 + \frac{Q_5}{\sqrt{Q_2Q_6}} dx_1^2 + \sqrt{\frac{Q_2}{Q_6}} \sum_{i=2}^{5} dx_i^2, \\
e^{-2\phi} &= \frac{Q_6^{\frac{3}{2}}}{Q_5\sqrt{Q_2}} , \;\; F_{\rho\tau y x_1} = \frac{\rho}{2Q_5Q_2}, \;\; H_{x_1\theta\phi} = -Q_5 \sin\theta, \;\; (dA)_{\theta\phi} = -Q_6 \sin\theta ,
\end{aligned} \tag{2.7}$$

where we have made also the coordinate transformations $\tau = 2\sqrt{Q_5}t$, $z = 2\sqrt{Q_5}y$, $r = \rho^2$. The above geometry is now the product of $S^2 \times S'^1 \times T^4$ with the Schwarzschild black hole in AdS_3.

3. Entropy function for non-extremal $D1D5$-branes

Following [4], in order to apply the entropy function formalism to the non-extremal $D1D5$-branes one should deform the near horizon geometry (2.4) to the most general form which is the product of the AdS-Schwarzchild and $S^3 \times T^4$ space, that is

$$\begin{aligned} ds_{10}^2 &= v_1 \left[\frac{r^2}{\sqrt{Q_1 Q_5}} \Big\{ -\left(1 - \frac{r_0^2}{r^2}\right) dt^2 + dz^2 \Big\} + \frac{\sqrt{Q_1 Q_5}}{r^2} \left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 \right] \\ &\quad + v_2 \left[\sqrt{Q_1 Q_5} (d\Omega_3)^2 + \sqrt{\frac{Q_1}{Q_5}} \sum_{i=1}^{4} dx_i^2 \right], \\ e^{-2\phi} &= \frac{Q_5}{Q_1} u \,, \qquad F_{rtz} = \frac{2r}{Q_1} \frac{v_1^{\frac{3}{2}}}{v_2^{\frac{7}{2}}} \equiv e_1 \,, \qquad F_{rtzx_1 \cdots x_4} = \frac{2r}{Q_5} v_1^{\frac{3}{2}} v_2^{\frac{1}{2}} \equiv e_2 \,, \end{aligned} \tag{3.1}$$

where v_1, v_2, u are supposed to be constants, otherwise the above geometry is not product space. The electric field strengths are deformed such that the electric charges are remaining fixed. The function f is defined to be the integral of the Lagrangian density over the horizon $H = S^1 \times S^3 \times T^4$. The result of inserting the background of (3.1) into f is

$$\begin{aligned} f(v_1, v_2, u, e_1, e_2, r) &\equiv \frac{1}{16\pi G_{10}} \int dx^H \sqrt{-g} \mathcal{L} \\ &= \frac{V_1 V_3 V_4 r}{16\pi G_{10}} Q_1^{3/2} Q_5^{-1/2} v_1^{3/2} v_2^{7/2} \\ &\quad \times \left(\frac{6u Q_5^{\frac{1}{2}} (v_1 - v_2)}{Q_1^{\frac{3}{2}} v_1 v_2} + \frac{Q_1^{\frac{1}{2}} Q_5^{\frac{1}{2}}}{2 v_1^3 r^2} e_1^2 + \frac{Q_5^{\frac{5}{2}}}{2 Q_1^{\frac{3}{2}} v_1^3 v_2^4 r^2} e_2^2 \right) , \end{aligned} \tag{3.2}$$

where V_1 is the volume of S^1, V_3 is the volume of the 3-sphere with radius one, and V_4 is the T^4 volume. The electric charges are carried by the branes and are given by

$$q_1 = \frac{\partial f}{\partial e_1} = \frac{V_1 V_3 V_4 Q_1^2 v_2^{\frac{7}{2}}}{16\pi G_{10} v_1^{\frac{3}{2}} r} e_1 \,, \qquad q_2 = \frac{\partial f}{\partial e_2} = \frac{V_1 V_3 V_4 Q_5^2}{16\pi G_{10} v_1^{\frac{3}{2}} v_2^{\frac{1}{2}} r} e_2 \,. \tag{3.3}$$

Note that the electric charges are independent of the scales v_1 and v_2 as expected, i.e.,

$$q_1 = \frac{V_1 V_3 V_4}{8\pi G_{10}} Q_1 \,, \qquad q_2 = \frac{V_1 V_3 V_4}{8\pi G_{10}} Q_5 \,. \tag{3.4}$$

Following [4], for AdS_2 space, one defins the entropy function as the Legendre transform of f with respect to the electric fields e_1 and e_2. Extending that definition to our case which is AdS_3 space, we define the entropy function by taking the Legendre transform of f with respect to the electric fields e_1 and e_2, and dividing the result by r, that is[2]

$$\begin{aligned} F(v_1, v_2, u) &\equiv \frac{1}{r} \left(e_1 \frac{\partial f}{\partial e_1} + e_2 \frac{\partial f}{\partial e_2} - f \right) \\ &= \frac{V_1 V_3 V_4}{16\pi G_{10}} v_1^{3/2} v_2^{7/2} \left(\frac{6u(v_2 - v_1)}{v_1 v_2} + \frac{2}{v_2^7} + \frac{2}{v_2^3} \right) , \end{aligned} \tag{3.5}$$

[2]For AdS_{2+p} space, one should divide the Legendre transform of f by r^p to define the entropy function in AdS_{2+p} space.

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where we have substituted the values of e_1 and e_2 from (3.1). Note that we have already assumed that v_1, v_2 and u are independent of r, that is, $\dot{v_1}$, $\dot{v_2}$ and $\dot{u}$ are not appeared in the Lagrangian. Hence, diving the Legendre transform of f by r does not change the equations of motion.[3] Solving the equations of motion

$$\frac{\partial F}{\partial v_i} = 0\,, \quad i = 1, 2\,; \qquad \frac{\partial F}{\partial u} = 0\,, \tag{3.6}$$

one finds the following solution

$$v_1 = 1\,, \; v_2 = 1\,, \; u = 1. \tag{3.7}$$

This confirms that (2.4) is a solution of the type IIB supergravity action.

Let us now consider the behavior of the entropy function around the above critical point. To this end, consider the following matrix

$$M_{ij} = \partial_i \partial_j F(v_1, v_2, u)\,. \tag{3.8}$$

Ignoring the overall factor, the eigenvalues of this matrix are $(68.10, -10.87, 0.78)$. This shows that the above critical point is a saddle point of the entropy function. It is a general property of the entropy function for both extremal and non-extremal cases [7].

Let us now return to the entropy associated with this solution. It is straightforward to find the entropy from the Wald formula [3]

$$S_{\rm BH} = -\frac{8\pi}{16\pi G_{10}} \int dx^H \sqrt{g^H} \frac{\partial \mathcal{L}}{\partial R_{trtr}} g_{tt} g_{rr}\,. \tag{3.9}$$

For this background we have $R_{trtr} = \frac{1}{v_1\sqrt{Q_1 Q_5}} g_{tt} g_{rr}$ and $\sqrt{-g} = v_1 \sqrt{g^H}$. These simplify the entropy relation to

$$S_{\rm BH} = -\frac{8\pi\sqrt{Q_1 Q_5}}{16\pi G_{10}} \int dx^H \sqrt{-g} \frac{\partial \mathcal{L}}{\partial R_{trtr}} R_{trtr} = -2\pi\sqrt{Q_1 Q_5} \frac{\partial f_\lambda}{\partial \lambda}\bigg|_{\lambda=1}\,, \tag{3.10}$$

where f_λ is an expression similar to f except that each R_{trtr} Riemann tensor component is scaled by a factor of λ.

To find $\frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1}$ using the prescription given in [4] and [5], we note that in addition to R_{trtr}, the Riemann tensor components R_{tztz} and R_{rzrz} are proportional to v_1, i.e.,

$$R_{trtr} = -\frac{v_1}{\sqrt{Q_1 Q_5}}\,, \qquad R_{rzrz} = \frac{v_1 r^2}{\sqrt{Q_1 Q_5}(r^2 - r_0^2)}\,, \qquad R_{tztz} = -\frac{v_1 r^2 (r^2 - r_0^2)}{(Q_1 Q_5)^{\frac{3}{2}}}\,. \tag{3.11}$$

Hence, one should rescale them too. We use the following scaling for these components

$$R_{tztz} \to \lambda_1 R_{tztz}\,, \qquad R_{rzrz} \to \lambda_2 R_{rzrz}\,. \tag{3.12}$$

[3] An alternative way to deal with the AdS_{2+p} space is to dimensionally reduce it to AdS_2 space and then use the entropy function formalism of the AdS_2 space [15].

Now, $f_\lambda(v_1, v_2, u, e_1, e_2)$ must be of the form $v_1^{\frac{3}{2}} g(v_2, \lambda v_1, \lambda_1 v_1, \lambda_2 v_1, e_1 v_1^{-\frac{3}{2}}, e_2 v_1^{-\frac{3}{2}})$ for some function g. Then one can show that the following relation holds for f_λ and its derivatives with respect to scales, λ_i, e_1, e_2 and v_1:

$$\lambda \frac{\partial f_\lambda}{\partial \lambda} + \lambda_1 \frac{\partial f_\lambda}{\partial \lambda_1} + \lambda_2 \frac{\partial f_\lambda}{\partial \lambda_2} + \frac{3}{2} e_1 \frac{\partial f_\lambda}{\partial e_1} + \frac{3}{2} e_2 \frac{\partial f_\lambda}{\partial e_2} + v_1 \frac{\partial f_\lambda}{\partial v_1} - \frac{3}{2} f_\lambda = 0 \,. \tag{3.13}$$

In addition, there is a relation between the rescaled Riemann tensor components at the supergravity level, which can be found by using (3.11)

$$\left.\frac{\partial f_\lambda}{\partial \lambda_1}\right|_{\lambda_1=1} + \left.\frac{\partial f_\lambda}{\partial \lambda_2}\right|_{\lambda_2=1} = 2 \left.\frac{\partial f_\lambda}{\partial \lambda}\right|_{\lambda=1} . \tag{3.14}$$

Replacing the above relation into (3.13) and using the equations of motion, one finds that $\frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1} = -\frac{r}{2} F$. It is easy to see that the entropy is proportional to the entropy function up to a constant coefficient, i.e.,

$$S_{\rm BH} = \pi \sqrt{Q_1 Q_2} r_0 F = \frac{V_1 V_3 V_4 r_0 \sqrt{Q_1 Q_5}}{4 G_{10}} \,, \tag{3.15}$$

This is the Bekenstein-Hawking entropy. One may write the entropy in terms of the temperature of black brane. The relation between r_0 and temperature can be read from the metric. The surface gravity is given by

$$\kappa = 2\pi T = \sqrt{g^{rr}} \frac{d}{dr} \sqrt{-g_{tt}} \Big|_H \tag{3.16}$$

which in our case we find $r_0 = 2\pi \sqrt{Q_1 Q_5} T$. Note that the constant v_1 is canceled in the above surface gravity. This causes that the higher derivative terms which modifies v_1 have no effect on the temperature. The entropy in terms of temperature becomes

$$S_{\rm BH} = 2\pi N_1 N_5 V_1 T \,, \tag{3.17}$$

where we have used the relations $V_3 = 2\pi^2$, $V_4 Q_1 = 16\pi^4 \alpha'^3 g_s N_1$, $Q_5 = \alpha' g_s N_5$, and $16\pi G_{10} = (2\pi)^7 \alpha'^4 g_s^2$ where N_1 is the number of D1-branes and N_5 is the number of D5-branes [5]. Alternatively, one may write the entropy in terms of the number of left moving or right moving momenta. Note that for our case $N_R = N_L$. The relation between r_0 and N_R is given as

$$N_R = \frac{r_0^2 (V_1/2\pi)^2 V_4/(2\pi)^4}{4 g_s^2 \alpha'^4}$$

where we have set $\sigma = 0$ in the relations for N_R and N_L in [14]. In terms of N_R, the entropy (3.15) becomes

$$\begin{aligned} S_{\rm BH} &= 4\pi \sqrt{N_1 N_5 N_R} \\ &= 2\pi \sqrt{N_1 N_5} \left(\sqrt{N_L} + \sqrt{N_R} \right) \end{aligned} \tag{3.18}$$

Note that for two charges extremal black hole, $r_0 = 0$, i.e., $N_R = N_L = 0$, the entropy function is exactly the same as the non-extremal case but the value of the entropy is zero.

We have seen that the entropy function works despite the fact that the horizon is not attractive. To see more explicitly that the horizon here is not attractive, we use the intuitional explanation for attractor mechanism given in [12]. According to this, the physical distance from an arbitrary point to the attractive horizon is infinite. The proper distance of an arbitrary point from the horizon in our case is

$$\rho = \int_{r_0}^{r} \frac{(Q_1Q_5)^{1/4}}{r}\left(1-\frac{r_0^2}{r^2}\right)^{-\frac{1}{2}} dr = (Q_1Q_5)^{1/4}\log\left[\frac{r}{r_0}+\sqrt{\frac{r^2}{r_0^2}-1}\right], \tag{3.19}$$

which is finite (infinite) for the non-extremal (extremal) case.

3.1 Higher derivative terms

In the previous sections we have seen that the entropy function formalism works at two derivatives level. It will be interesting to consider stringy effects and take a look at the entropy function mechanism again. To this end, we consider the higher derivative corrections coming from string theory. To next leading order the Lagrangian of type II theory is given by [17]

$$S = \frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{-g}\left\{\mathcal{L}^{\text{tree}} + e^{-2\phi}\left(\gamma W\right)\right\}, \tag{3.20}$$

where $\gamma = \frac{1}{8}\zeta(3)(\alpha')^3$ and W can be written in terms of the Weyl tensors

$$W = C^{hmnk}C_{pmnq}C_h{}^{rsp}C^q{}_{rsk} + \frac{1}{2}C^{hkmn}C_{pqmn}C_h{}^{rsp}C^q{}_{rsk}. \tag{3.21}$$

Following [4], we consider the general background consist of AdS-Schwarzchild times $S^3\times T^4$ space (3.1) in the presence of the higher derivative terms. As we shall see shortly, the higher derivative terms respect the symmetry of the tree level solution, i.e., the coefficients v_1 and v_2 remain constant. To see this we calculate the contribution of the above higher derivative terms to the entropy function[4]

$$\begin{aligned}\delta F &= -\frac{\gamma Q_5 u}{16\pi G_{10} r Q_1}\int dx^H\sqrt{-g}W \\ &= -\gamma u\frac{V_1V_3V_4\sqrt{Q_1Q_5}}{16\pi G_{10}}v_1^{\frac{3}{2}}v_2^{\frac{7}{2}}\left[\frac{105(v_2^4-\frac{4}{7}v_1^3v_2+\frac{18}{35}v_1^2v_2^2-\frac{4}{7}v_1v_2^3+v_1^4)}{32Q_1^2Q_5^2v_1^4v_2^4}\right].\end{aligned} \tag{3.22}$$

It is important to note that δF is independent of r. This is consistent with our assumption that v_1, v_2 and u are constants. By variation of $F+\delta F$ with respect to v_1, v_2 and u one finds the equations of motion. Since these equations are valid only up to first order of γ, we consider the following perturbative solutions:

$$v_1 = 1+\gamma x, \quad v_2 = 1+\gamma y, \quad u = 1+\gamma z. \tag{3.23}$$

[4]Note that for $AdS_3\times S^3$ with identical radii, the Weyl tensor is zero as noted in [16]. However, this tensor is non-vanishing for the space $AdS_3\times S^3\times T^4$ in which we are interested in 10-dimensional space-time.

One should replace them into the equations of motion, i.e.,

$$\begin{aligned}
\frac{\partial(F+\delta F)}{\partial u} &= 0 \longrightarrow \quad 6(y-x) = \frac{9}{2(Q_1Q_5)^{\frac{3}{2}}}\,, \\
\frac{\partial(F+\delta F)}{\partial v_1} &= 0 \longrightarrow \quad 28y+4x+8z = \frac{3}{(Q_1Q_5)^{\frac{3}{2}}}\,, \\
\frac{\partial(F+\delta F)}{\partial v_2} &= 0 \longrightarrow \quad -244y+84x-24z = -\frac{27}{(Q_1Q_5)^{\frac{3}{2}}}\,,
\end{aligned} \tag{3.24}$$

these equations are consistent and give the following results:

$$v_1 = 1-\gamma\frac{51}{32(Q_1Q_5)^{\frac{3}{2}}}\,,\ v_2 = 1-\gamma\frac{27}{32(Q_1Q_5)^{\frac{3}{2}}}\,,\ u = 1+\gamma\frac{33}{8(Q_1Q_5)^{\frac{3}{2}}}\,. \tag{3.25}$$

It is interesting to note that the stringy effect decreases the closed string coupling at the near horizon, i.e., $\phi = \phi_0 - 33\gamma/[16(Q_1Q_5)^{3/2}]$. Similar behavior appears for the non-extremal $D3$-branes [16].

Let us now return to the entropy associated with this solution. The entropy is given by

$$S_{\rm BH} = \pi\sqrt{Q_1Q_5}\,r_0(F+\delta F)\,, \tag{3.26}$$

where we have used the fact that all the steps toward writing the Wald formula (3.10) for entropy in terms of the above entropy function remain unchanged. In particular the relation (3.14) holds in the presence of the higher derivative terms. It turns out, in order to find the entropy to linear order of γ, one does not need to know the values of x, y, and z. To see this, note that if one replaces (3.23) into the first term above, one finds that x, y, and z do not appear in this term linearly. The second term has an overall factor of γ, hence to the linear order of γ, one has to replace $v_1 = v_2 = u = 1$ into it. The result is

$$S_{\rm BH} = \frac{V_1V_3V_4r_0\sqrt{Q_1Q_5}}{4G_{10}}\left[1-\gamma\frac{9}{8(Q_1Q_5)^{3/2}}+O(\gamma^2)\right]. \tag{3.27}$$

As a double check, we calculate the entropy using the ward formula (3.10) directly, i.e.,

$$S_{\rm BH} = -2\pi\sqrt{Q_1Q_5}\left(\left.\frac{\partial f_\lambda}{\partial\lambda}\right|_{\lambda=1}+\left.\frac{\partial f^W_\lambda}{\partial\lambda}\right|_{\lambda=1}\right)\,, \tag{3.28}$$

where the function f^W is given by

$$f^W = \frac{\gamma}{16\pi G_{10}}\int dx^H\sqrt{-g}e^{-2\phi}W\,. \tag{3.29}$$

This second term is proportional to γ, so to the first order of γ one has to replace the Schwarzschild AdS solution (2.4) in $\partial f^W_\lambda/\partial\lambda$ which gives

$$\left.\frac{\partial f^W_\lambda}{\partial\lambda}\right|_{\lambda=1} = \gamma\frac{V_1V_3V_4r}{16\pi G_{10}}\left[\frac{3}{(Q_1Q_5)^{3/2}}\right]. \tag{3.30}$$

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For the first term, on the other hand, one has to replace (3.23) which gives

$$\left.\frac{\partial f_\lambda}{\partial \lambda}\right|_{\lambda=1} = \frac{V_1 V_3 V_4 r}{16\pi G_{10}}\left[-2-\gamma\frac{7y+x+2z}{(Q_1Q_5)^{3/2}}\right]. \tag{3.31}$$

Now inserting the solution (3.25) for x, y and z into the above equation, one finds exactly the result (3.27).

To write the entropy in terms of the temperature, we note that v_1 appears as an overall factor of AdS_3 in the background (3.1), hence, the temperature (3.16) remains the same as the tree level temperature, i.e., $r_0 = 2\pi\sqrt{Q_1Q_5}T$. This is unlike the temperature of non-extremal $D3$-branes that stringy effects increase the tree level temperature.

The entropy of $D1D5$-branes in terms of temperature or in terms of N_R is

$$\begin{aligned} S_{\rm BH} &= 2\pi N_1 N_5 V_1 T\left[1-\gamma\frac{9}{8}\left(\frac{(2\pi)^3V_4}{16\pi G_{10}N_1N_5}\right)^{3/2}+O(\gamma^2)\right] \\ &= 4\pi\sqrt{N_1N_5N_R}\left[1-\gamma\frac{9}{8}\left(\frac{(2\pi)^3V_4}{16\pi G_{10}N_1N_5}\right)^{3/2}+O(\gamma^2)\right]. \end{aligned} \tag{3.32}$$

In the second line, we have used the fact that the higher derivative corrections do not change the Hawking temperature which means the number of excitations for the left and right moving momenta remain the same as the tree level result. Note that the leading α' correction makes the entropy decreases.

4. Entropy function for non-extremal $D2D6NS5$-branes

Following [4], in order to apply the entropy function formalism to the non-extremal $D2D6NS5$-branes one should deform the near horizon geometry (2.7) to the most general form which is the product of the AdS-Schwarzchild and $S'^1\times S^2\times T^4$ space, that is

$$\begin{aligned} ds_{10}^2 &= v_1\left[\frac{\rho^2}{4Q_5\sqrt{Q_2Q_6}}\left\{-\left(1-\frac{\rho_0^2}{\rho^2}\right)d\tau^2+dy^2\right\}+\frac{4Q_5\sqrt{Q_2Q_6}}{\rho^2}\left(1-\frac{\rho_0^2}{\rho^2}\right)^{-1}d\rho^2\right] \\ &\quad +v_2\left[Q_5\sqrt{Q_2Q_6}(d\Omega_2)^2+\frac{Q_5}{\sqrt{Q_2Q_6}}dx_1^2+\sqrt{\frac{Q_2}{Q_6}}\sum_{i=2}^5 dx_i^2\right], \\ F_{\rho\tau y x_1} &= \frac{\rho}{2Q_5Q_2}\frac{v_1^{\frac{3}{2}}}{v_2^{\frac{5}{2}}}\equiv e_1\,, \qquad H_{x_1\theta\phi}=-Q_5\sin\theta\,, \qquad (dA)_{\theta\phi}=-Q_6\sin\theta\,, \\ e^{-2\phi} &= \frac{Q_6^{\frac{3}{2}}}{Q_5\sqrt{Q_2}}u\,, \end{aligned} \tag{4.1}$$

where v_1, v_2, u are supposed to be constants. The electric field strength is deformed such that the corresponding electric charge remains fixed. Similarly, to have the fixed magnetic charges, one does not need to deform the magnetic field strength. The function f is defined to be the integral of the Lagrangian density over the horizon $H = S^1 \times S'^1 \times S^2 \times T^4$. The result of inserting the background (4.1) into f is

$$\begin{aligned} f(v_1, v_2, , u, e_1) &\equiv \frac{1}{16\pi G_{10}} \int dx^H \sqrt{-g}\mathcal{L} \\ &= \frac{V_1 V_1' V_2 V_4 \rho}{32\pi G_{10}} Q_2 Q_5 Q_6^{-1} v_1^{3/2} v_2^{7/2} \\ &\times \left(\frac{u Q_6 (4v_1 - 3v_2)}{2Q_2 Q_5^2 v_1 v_2} + \frac{2Q_2 Q_6}{v_1^3 v_2 \rho^2} e_1^2 - \frac{Q_6}{2v_2^2 Q_5^2 Q_2} - \frac{Q_6 u}{2v_2^3 Q_5^2 Q_2} \right) , \end{aligned} \tag{4.2}$$

where $V_1(V_1')$ is the volume of $S^1(S'^1)$, V_2 is the volume of the 2-sphere with radius one, and V_4 is the T^4 volume. The electric charge carried by the $D2$-brane is given by

$$q_1 = \frac{\partial f}{\partial e_1} = \frac{V_1 V_1 V_2 V_4 Q_2^2 Q_5 v_2^{\frac{5}{2}}}{8\pi G_{10} v_1^{\frac{3}{2}} \rho} e_1 . \tag{4.3}$$

Note that the electric charge is independent of the scales v_1, v_2 as expected, i.e.,

$$q_1 = \frac{V_1 V_1 V_2 V_4}{16\pi G_{10}} Q_2 . \tag{4.4}$$

Now we define the entropy function by taking the Legendre transform of f with respect to the electric field e_1, and dividing by ρ, that is

$$\begin{aligned} F(v_1, v_2, u) &\equiv \frac{1}{\rho} \left(e_1 \frac{\partial f}{\partial e_1} - f \right) \\ &= \frac{V_1 V_1 V_2 V_4}{32\pi G_{10} Q_5} v_1^{3/2} v_2^{7/2} \left(\frac{u(3v_2 - 4v_1)}{2v_1 v_2} + \frac{1}{2v_2^6} + \frac{1}{2v_2^2} + \frac{u}{2v_2^3} \right) , \end{aligned}$$

where we have substituted the value of e_1. Solving the equations of motion

$$\frac{\partial F}{\partial v_i} = 0 , \quad i = 1, 2 ; \qquad \frac{\partial F}{\partial u} = 0 , \tag{4.5}$$

one finds the following solutions

$$v_1 = 1 , \; v_2 = 1 , \; u = 1 . \tag{4.6}$$

This confirms that (2.7) is a solution of the type IIA supergravity action. To find the behavior of the entropy function around the above critical point, consider again the matrix (3.8). Ignoring the overall factor, the eigenvalues of this matrix are $(12.44, -3.30, 0.37)$. This shows again that the above critical point is a saddle point of the entropy function.

Let us now return to the entropy associated with this solution. The Wald formula [3] is given by

$$S_{\rm BH} = -\frac{8\pi}{16\pi G_{10}} \int dx^H \sqrt{g^H} \frac{\partial \mathcal{L}}{\partial R_{\tau\rho\tau\rho}} g_{\tau\tau} g_{\rho\rho} . \tag{4.7}$$

For this background we have $R_{\tau\rho\tau\rho} = \frac{1}{4v_1 Q_5\sqrt{Q_2Q_6}} g_{\tau\tau}g_{\rho\rho}$ and $\sqrt{-g} = v_1\sqrt{g^H}$. These simplify the entropy relation to

$$S_{\rm BH} = -\frac{32\pi Q_5\sqrt{Q_2Q_6}}{16\pi G_{10}}\int dx^H\sqrt{-g}\frac{\partial\mathcal{L}}{\partial R_{\tau\rho\tau\rho}}R_{\tau\rho\tau\rho} = -8\pi Q_5\sqrt{Q_2Q_6}\frac{\partial f_\lambda}{\partial\lambda}\bigg|_{\lambda=1}, \tag{4.8}$$

where f_λ is an expression similar to f except that each $R_{\tau\rho\tau\rho}$ Riemann tensor component is scaled by a factor of λ.

To find $\frac{\partial f_\lambda}{\partial\lambda}|_{\lambda=1}$, we note that in addition to $R_{\tau\rho\tau\rho}$, the Riemann tensor components $R_{\tau y\tau y}$ and $R_{\rho y\rho y}$ are proportional to v_1, i.e.,

$$R_{\tau\rho\tau\rho} = -\frac{v_1}{4Q_5(Q_2Q_6)^{\frac{1}{2}}}\,,\; R_{\rho y\rho y} = \frac{v_1\rho^2}{4Q_5(Q_2Q_6)^{\frac{1}{2}}(\rho^2-\rho_0^2)}\,,\; R_{\tau y\tau y} = -\frac{v_1\rho^2(\rho^2-\rho_0^2)}{64Q_5^3(Q_2Q_6)^{\frac{3}{2}}}\,. \tag{4.9}$$

Hence, one should also rescale these components. We use the following scaling

$$R_{\tau y\tau y}\to\lambda_1 R_{\tau y\tau y}\,,\qquad R_{\rho y\rho y}\to\lambda_2 R_{\rho y\rho y}\,. \tag{4.10}$$

Now we see that $f_\lambda(v_1,v_2,u,e_1)$ must be of the form $v_1^{\frac{3}{2}}g(v_2,\lambda v_1,\lambda_1 v_1,\lambda_2 v_1,e_1 v_1^{-\frac{3}{2}})$ for some function g. Then one can show that the following relation holds for f_λ and its derivatives with respect to scales, λ_i, e_1 and v_1

$$\lambda\frac{\partial f_\lambda}{\partial\lambda}+\lambda_1\frac{\partial f_\lambda}{\partial\lambda_1}+\lambda_2\frac{\partial f_\lambda}{\partial\lambda_2}+\frac{3}{2}e_1\frac{\partial f_\lambda}{\partial e_1}+v_1\frac{\partial f_\lambda}{\partial v_1}-\frac{3}{2}f_\lambda = 0\,. \tag{4.11}$$

As in the $D1D5$ case, by using the equation (4.9) one finds the same relation as (3.14) between the rescaled Riemann tensor components at the supergravity level. Replacing (3.14) into (4.11) and using the equations of motion, one finds again $\frac{\partial f_\lambda}{\partial\lambda}|_{\lambda=1} = -\frac{\rho}{2}F$. Hence, the entropy is proportional to the entropy function up to a constant coefficient, i.e.,

$$S_{\rm BH} = 4\pi Q_5\sqrt{Q_2Q_6}\rho_0 F = \frac{V_1V_1'V_2V_4\rho_0\sqrt{Q_2Q_6}}{8G_{10}}\,, \tag{4.12}$$

This is the Bekenstein-Hawking entropy. One may write the entropy in terms of temperature. An alternative way to find temperature is to impose regularity of Euclidean metric near the horizon. So consider the proper distance of an arbitrary point from the horizon, i.e.,

$$r = \int_{\rho_0}^{\rho}\frac{2(v_1Q_5)^{\frac{1}{2}}(Q_2Q_6)^{\frac{1}{4}}}{\rho}\left(1-\frac{\rho_0^2}{\rho^2}\right)^{-\frac{1}{2}}d\rho = 2(v_1Q_5)^{\frac{1}{2}}(Q_2Q_6)^{\frac{1}{4}}\log\left[\frac{\rho}{\rho_0}+\sqrt{\frac{\rho^2}{\rho_0^2}-1}\right]. \tag{4.13}$$

Near ρ_0, one finds $\rho^2 = \rho_0^2(1+r^2/4v_1Q_5\sqrt{Q_2Q_6})$. So the metric (4.1) near ρ_0 becomes

$$ds^2 = -\frac{\rho_0^2}{16Q_5^2Q_2Q_6}r^2d\tau^2+dr^2+\cdots\,. \tag{4.14}$$

The period of the Euclidean time, required by the regularity of metric is $1/T = \beta = 8\pi Q_5\sqrt{Q_2Q_6}/\rho_0$. Note that here also the constant v_1 does not appear in the above metric, so the temperature is independent of the value of v_1. The entropy in terms of the temperature is

$$S_{\rm BH} = \frac{\pi}{G_{10}} V_1 V_1' V_2 V_4 Q_2 Q_5 Q_6 T = 2\pi N_2 N_5 N_6 V_1 T\,, \tag{4.15}$$

where in the last expression we have used

$$\begin{aligned}
N_2 &= \frac{1}{\sqrt{16\pi G_{10}}\mu_2} \int_{S^2\times T^4} *F_{(4)} = \frac{Q_2 V_2 V_4}{16\pi G_{10} T_2}\,,\\
N_5 &= \frac{g_s}{\sqrt{16\pi G_{10}}\mu_5} \int_{S^2\times S'^1} H_{(3)} = \frac{g_s Q_5 V_1' V_2}{16\pi G_{10} T_5}\,,\\
N_6 &= \frac{1}{\sqrt{16\pi G_{10}}\mu_6} \int_{S^2} F_{(2)} = \frac{Q_6 V_2}{16\pi G_{10} T_6}\,,
\end{aligned} \tag{4.16}$$

where $\mu_p = \sqrt{16\pi G_{10}} T_p$ and $T_p = 2\pi/((2\pi\ell_s)^{p+1} g_s)$. Alternatively, one may write the entropy in terms of the number of left moving or right moving momenta where in our case $N_R = N_L$. The relation between ρ_0 and N_R is given as

$$N_R = \frac{\rho_0^2 (V_1/4\pi\sqrt{Q_5})^2 V_4/(2\pi)^4 V_1'/2\pi}{2 g_s^2 \alpha'^4}$$

where we have set $\alpha_p = 0$ in the relations for N_R and N_L in [14], and used the rescaling $z = 2\sqrt{Q_5} y$, $r_0 = \rho_0^2$. In terms of N_R, the entropy (4.12) becomes

$$\begin{aligned}
S_{\rm BH} &= 4\pi\sqrt{N_2 N_6 N_5 N_R}\\
&= 2\pi\sqrt{N_2 N_6 N_5}\left(\sqrt{N_L} + \sqrt{N_R}\right)
\end{aligned} \tag{4.17}$$

which is in the conventional form appearing in [14].

4.1 Higher derivative terms

We now consider the general background consist of AdS-Schwarzchild times $S'^1 \times S^2 \times T^4$ space (4.1) in the presence of the higher derivative terms. The higher derivative terms respect the symmetry of the tree level solution, i.e., the coefficients v_1 and v_2 remain constant. To see this we calculate the contribution of the above higher derivative terms to the entropy function i.e.,

$$\begin{aligned}
\delta F &= -\frac{\gamma Q_6^{\frac{3}{2}} u}{16\pi G_{10} Q_5 Q_2^{\frac{1}{2}} \rho} \int dx^H \sqrt{-g} W \qquad\qquad (4.18)\\
&= -\frac{\gamma u V_1 V_1' V_2 V_4 (Q_2 Q_6)^{\frac{1}{2}} v_1^{\frac{3}{2}} v_2^{\frac{7}{2}}}{32\pi G_{10}} \left[\frac{35(-\frac{3}{28} v_1^3 v_2 + \frac{81}{2048} v_2^4 + \frac{27}{224} v_2^2 v_1^2 - \frac{27}{896} v_2^3 v_1 + v_1^4)}{108 v_1^4 v_2^4 (Q_2 Q_6)^2 Q_5^4}\right],
\end{aligned}$$

By variation of $F + \delta F$ with respect to v_1, v_2 and u one finds the equations of motion. Considering the perturbative solutions (3.23), one finds

$$\begin{aligned}
\frac{\partial(F+\delta F)}{\partial u} &= 0 \longrightarrow\ \ y - 3x = \frac{73315}{110592 (Q_2 Q_5^2 Q_6)^{\frac{3}{2}}}\,,\\
\frac{\partial(F+\delta F)}{\partial v_1} &= 0 \longrightarrow\ \ 7y + x + 2z = -\frac{7075}{12288 (Q_2 Q_5^2 Q_6)^{\frac{3}{2}}}\,,\\
\frac{\partial(F+\delta F)}{\partial v_2} &= 0 \longrightarrow\ \ 41y - 21x + 2z = -\frac{44395}{110592 (Q_2 Q_5^2 Q_6)^{\frac{3}{2}}}\,,
\end{aligned} \tag{4.19}$$

these equations are consistent, and give the following results

$$
\begin{aligned}
v_1 &= 1-\gamma\frac{247343}{884736(Q_2Q_5^2Q_6)^{\frac{3}{2}}}\,, \quad v_2 = 1-\gamma\frac{155509}{884736(Q_2Q_5^2Q_6)^{\frac{3}{2}}}\,,\\
u &= 1+\gamma\frac{45917}{98304(Q_2Q_5^2Q_6)^{\frac{3}{2}}}\,.
\end{aligned} \tag{4.20}
$$

Similar to the $D1D5$ case, the stringy effects decrease the closed string coupling at the near horizon. Let us return to the entropy associated with this solution. The entropy is given by

$$
S_{\rm BH} = 4\pi Q_5\sqrt{Q_2Q_6}\rho_0(F+\delta F)\,, \tag{4.21}
$$

where again we have used the fact that all the steps toward writing the Wald formula for the entropy in terms of the entropy function above, remain unchanged. In this case also, in order to find the entropy to linear order of γ, one does not need to know the solutions for x, y and z. That is, if one replaces (3.23) into the tree level entropy function, i.e., the first term above, one finds that x, y, and z do not appear in it linearly. The second term, on the other hand, has an overall factor of γ, hence to linear order of γ, one has to replace $v_1 = v_2 = u = 1$ into it. The result is

$$
S_{\rm BH} = \frac{V_1V_1'V_2V_4\rho_0(Q_2Q_6)^{\frac{1}{2}}}{8G_{10}}\left[1-\gamma\frac{73315}{221184(Q_2Q_5^2Q_6)^{\frac{3}{2}}}+O(\gamma^2)\right]\,, \tag{4.22}
$$

As a double check, we calculate the entropy using the ward formula (4.8) directly, i.e.,

$$
S_{\rm BH} = -8\pi Q_5\sqrt{Q_2Q_6}\left(\frac{\partial f_\lambda}{\partial\lambda}\bigg|_{\lambda=1}+\frac{\partial f_\lambda^W}{\partial\lambda}\bigg|_{\lambda=1}\right)\,. \tag{4.23}
$$

The second term is proportional to γ, so to the first order of γ one has to replace the Schwarzschild AdS solution (2.7) in $\partial f_\lambda^W/\partial\lambda$ which gives

$$
\frac{\partial f_\lambda^W}{\partial\lambda}\bigg|_{\lambda=1} = \gamma\frac{V_1V_1'V_2V_4\rho}{16\pi G_{10}Q_5}\left[\frac{1205}{110592(Q_2Q_5^2Q_6)^{\frac{3}{2}}}\right]\,. \tag{4.24}
$$

For the first term, one has to replace (3.23) which gives

$$
\frac{\partial f_\lambda}{\partial\lambda}\bigg|_{\lambda=1} = \frac{V_1V_1'V_2V_4\rho}{16\pi G_{10}Q_5}\left[-\frac{1}{4}-\gamma\frac{7y+x+2z}{8(Q_2Q_5^2Q_6)^{\frac{3}{2}}}\right]\,. \tag{4.25}
$$

Now inserting the solutions for x, y and z into the above equation, one finds exactly the result (4.22). The entropy (4.22) in terms of temperature is

$$
S_{\rm BH} = 2\pi N_2N_5N_6V_1T\left[1-\gamma\frac{73315}{221184(Q_2Q_5^2Q_6)^{3/2}}+O(\gamma^2)\right]\,. \tag{4.26}
$$

This entropy, like the entropy of the $D1D5$-branes, is less than the Bekenstein-Hawking entropy. This is unlike the entropy of the non-extremal $D3$-branes [16] which is $S_{\rm BH}^{D3} =$

$\frac{\pi^2}{2}N^2V_3T^3(1+15\gamma+O(\gamma^2))$, where the first term is the Bekenstein-Hawking entropy and the second term which is the α' correction, is positive.

The increase in the entropy for $D3$-branes is consistent with the fact that the Bekenstein-Hawking entropy at strong 't Hooft coupling is less than the entropy of $N=4$ SYM theory at zero coupling by a factor of 3/4 [18]. On the other hand, the correction to the entropy at weak coupling is negative [19] which is an indication of smooth interpolation between the weak and strong coupling regimes. For $D1D5$-branes, our result indicates that the correction to the entropy at strong coupling is negative. On the other hand, it is known that the entropy at zero coupling is the same as the Bekenstein-Hawking entropy at strong coupling [20]. This indicates that the correction to the entropy at weak coupling should be non-vanishing too. Assuming the interpolating function between the strong and the weak coupling regimes of the Higgs branch of the $N=(4,4)$ SYM at finite temperature in $1+1$ dimensions does not cross the zeroth order entropy at any point in finite coupling, one expects the correction to the entropy at weak coupling to be negative. It would be interesting to perform this calculation.

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